

## MODELLING FLUID FLOW IN FRACTURED-POROUS ROCK MASSES BY FINITE-ELEMENT TECHNIQUES\*

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### SUMMARY

One of the major difficulties of modelling fluid flow processes in hard-rock geologies is the complex nature of the porosity systems. Hydraulic behaviour in these rock masses is characterized by both porous and fractured interflow zones. Traditionally, fractured-porous rocks have been modelled as an equivalent porous medium or as a system of fractures separated by impermeable blocks. A new method is proposed that unifies these two approaches for modelling fluid flow processes in fractured-porous media. The basic idea is to use a combination of isoparametric elements for the porous zones and line elements for the fractures. The coupling between the governing equations for each element type is achieved using the superposition principle. The effectiveness of the new approach is demonstrated by comparing numerical solutions with known solutions for problems of flow and solute transport in fractured-porous media.

### INTRODUCTION

One of the problems currently confronting the commercial nuclear power industry in the United States and other countries is that technology for permanent disposal of nuclear waste has not as yet been demonstrated. In recent years, a broad consensus has developed that the most feasible solution is disposal of the nuclear waste in deep geologic formations. In the United States, researchers at various laboratories are investigating a number of geologic rock types to determine their suitability for deep underground repositories that would be used to isolate future inventories of nuclear waste. One of the primary research projects is studying the extensive basalt formations that underlie the Hanford Site in southeastern Washington.<sup>1</sup>

Mathematical models play an important role in the process of evaluating the waste-isolation capability of a candidate geology, by providing a basis to quantify repository performance. Predictions of basic hydrologic parameters, such as groundwater pathlines and traveltimes, are required to determine the degree of waste isolation achieved by the geologic system.<sup>2</sup> In a hard-rock geology such as basalt, the problem of modelling fluid flow processes is complicated by the fact that the rock mass is characterized by both fractured and porous rock strata. The contrasting hydraulic properties of these strata determine the rate and direction of groundwater movement.

The conceptual approaches currently used in modelling fluid flow in fractured media

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generally fall into two 'distinct' categories: the continuum approach and the discontinuum or discrete-fracture approach. The first approach is based on the concept that a rock mass is modelled at a sufficiently large space scale<sup>3</sup> so that the idealization of an equivalent porous medium<sup>4,5</sup> is valid. The second approach is based on the concept that the rock mass is composed of impermeable blocks, separated by systems of fractures that are idealized as networks of planar conduits.<sup>6,7</sup> These two modelling approaches, as used in the past, have not been applicable to systems where flow through both porous strata and discrete fractures occurs simultaneously.

In this paper, we present a new approach to the problem of modelling fluid flow processes in fractured-porous media, which uses finite-element techniques. This new approach unifies both the continuum and discontinuum modelling approaches. Results from recent research efforts on the development of a two-dimensional finite-element model for fluid flow processes in fractured-porous media are presented and discussed.

## MODELLING APPROACH

### *Physical basis*

The geologic environment in the vicinity of a nuclear waste repository is expected to be perturbed by various processes associated with the properties and presence of the deep underground facility. With regard to the hydrologic regime, perhaps the most significant factor will be the radiogenic heat produced by the waste form.<sup>8</sup> Groundwater movement through the surrounding rock mass will respond to the thermal driving force (i.e. buoyancy) and to the changes in the fluid properties. The importance of these effects is related to the fact that the temperature perturbations around the repository may persist for as long as 10,000 yr.<sup>2</sup>

To realistically describe groundwater flow in a fractured-porous medium, one must first relate the major features of the rock mass to hydraulic behaviour. In the case of an extrusive igneous rock such as basalt, individual basalt flows generally exhibit three distinct intraflow structures: the top of the rock layer is effectively a porous feature, whereas the centre and the base of the rock layer are low-permeability zones consisting of systems of cooling fractures (i.e. fractures created when the lava was formed). The conceptual understanding of the basic features of the rock mass and the important driving forces leads to a modelling approach for non-isothermal groundwater flow that considers: (1) flow through porous zones, (2) flow through systems of fractures, (3) flow exchanges between primary pores and fractures, (4) heat transfer through the combined water-rock systems, and (5) coupling between fluid flow and heat transport.

In general, the modelling of solute transport through a fractured-porous medium requires the consideration of: (1) advection, (2) dispersion/diffusion, (3) sorption, and (4) decay processes in both porous and fracture zones. In low-permeability rocks, however, solute transport is often characterized by advection and dispersion in the fractures and molecular diffusion through the so-called porous zones.<sup>9</sup> In a rock mass that exhibits large contrasts between the total and effective porosities, molecular diffusion into and out of dead-end pores can have a significant effect on the rate of solute transport. This secondary diffusion effect represents a physical retardation that has important implications to nuclear waste isolation in hard-rock geologies.<sup>10</sup>

### *Governing equations*

The mathematical models for simultaneous fluid flow and heat transport are based on the general principles of fluid continuity and energy conservation.<sup>11</sup> The fluid flow equations are

specialized to a fractured-porous medium by introducing the applicable flow law. In a porous medium, Darcy's flow law applies, whereas in individual fractures, the Poiseuille equation for laminar flow in planar conduits is applicable. These linear flow laws are very similar and differ only in the formulation of the hydraulic conductivity. The fundamental flow law can be concisely written in indicial notation as:

$$q_i = -K_{ii} \left( \frac{\partial h}{\partial x_i} + \delta_b \delta_{i3} \right) \quad (1)$$

where

- $q_i$  = fluid velocity components
- $K_{ii}$  = principal hydraulic conductivity components
- $h$  = hydraulic head
- $\delta_b$  = density disparity
- $\delta_{i3}$  = Kronecker delta.

In the above equation, repeated indices imply summation. The density disparity is a function of fluid density and is computed from:

$$\delta_b = \frac{\rho - \rho_0}{\rho^*} \quad (2)$$

In this expression

- $\rho$  = fluid density
- $\rho_0$  = initial fluid density, which may vary with depth
- $\rho^*$  = a constant fluid density value corresponding to a particular reference temperature.

For the continuum portion of the rock mass, the groundwater flow equation for non-isothermal flow is:

$$S_s \frac{\partial h}{\partial t} = K_{ii} \frac{\partial}{\partial x_i} \left( \frac{\partial h}{\partial x_i} + \delta_b \delta_{i3} \right) + \gamma \frac{\partial T}{\partial t} \quad (3)$$

where

- $S_s$  = specific storage
- $\gamma$  = thermal coupling coefficient
- $T$  = temperature of the water-rock system
- $t$  = time.

For the discrete fractures, the mathematical model for flow in the fractures is a one-dimensional equation expressed as:

$$S_f \frac{\partial h}{\partial t} = K_f \frac{\partial}{\partial L} \left( \frac{\partial h}{\partial L} + \delta'_b \right) + \gamma \frac{\partial T}{\partial t} \quad (4)$$

where

- $S_f$  = specific storage in the fracture
- $L$  = co-ordinate along the fracture
- $K_f$  = hydraulic conductivity of the filled or unfilled fracture
- $\delta'_b$  = component of the density disparity along  $L$ .

The time-dependence of the hydraulic head in the fracture enters primarily through the thermal coupling and the interactions with the flow in the continuum portion of the rock mass. For unfilled fractures, the hydraulic conductivity<sup>3</sup> is computed from:

$$K_f = \frac{ge^2}{12\nu} \quad (5)$$

where

- $g$  = acceleration of gravity
- $e$  = fracture aperture
- $\nu$  = kinematic viscosity of the fluid.

Under the assumption that the fluid and rock mass are in thermal equilibrium,<sup>12</sup> the thermal energy balance on the water-rock system yields a single governing equation for heat transport, written as:

$$S_t \frac{\partial T}{\partial t} + \rho c_f q_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} \left( D_t \frac{\partial T}{\partial x_i} \right) + Q \quad (6)$$

where

$S_t$  = heat capacity of the water-rock system

$c_f$  = specific heat of the fluid

$D_t$  = thermal conductivity of the water-rock system

$Q$  = heat-generation rate.

It is generally found, in a rock with low porosity and permeability, that the advective component is very small, so that the dominant mode of heat transport in the water-rock system is by pure conduction.

For a single component, the general form of the solute transport equation is expressed by:

$$R_d \phi \frac{\partial C}{\partial t} + q_i \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( D_m \phi \frac{\partial C}{\partial x_i} \right) - \lambda R_d \phi C + \dot{m} \quad (7)$$

where

$C$  = concentration

$R_d$  = retardation factor

$\phi$  = effective porosity

$D_m$  = mass dispersion (hydrodynamic) coefficient

$\lambda$  = decay constant

$\dot{m}$  = mass source term.

In the porous zones, the governing equation applies as stated above, whereas in the discrete fractures it simplifies to a one-dimensional form and the porosity is unity for the unfilled fractures.

## FINITE-ELEMENT TECHNIQUES

### *Methods of weighted residuals*

The governing equations for fluid flow in the porous continuum and discrete fractures, as presented earlier, were formulated for each distinct flow system. The coupling between the two systems is achieved by using the principle of superposition. In essence, the approach consists of summing the Galerkin functional equations for the porous continuum and the discrete fracture elements. Mathematically, the superposition is represented by:

$$\chi = \int_{\bar{R}} \omega_j \epsilon^c dR + \int_{\bar{L}} \omega_j \epsilon^d dL \quad (8)$$

where

$\epsilon^c$  and  $\epsilon^d$  = residual error vectors for the continuum elements  
and discrete fractures, respectively

$\omega_j$  = set of weighting functions

$\bar{R}$  = continuum domain

$\bar{L}$  = discrete domain.

The finite-element representation of the space domain consists of two-dimensional isoparametric elements, with line elements embedded along the sides of the two-dimensional

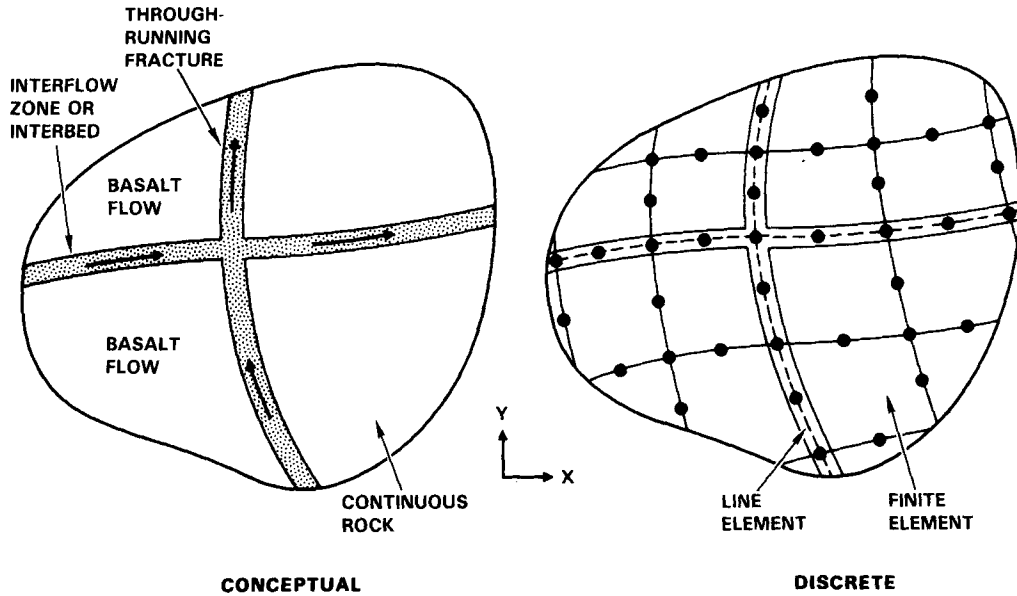


Figure 1. Finite-element representation of continuum and discrete features

elements (Figure 1). Since the two-dimensional and line elements share the same set of nodes, flow and mass continuity between the two systems are enforced.

The residual error vectors in the functional equations are expanded, using a Newton-Raphson approximation,<sup>13</sup> to formulate the finite-element equations in terms of the incremental changes in the dependent variables,  $\Delta T$ ,  $\Delta h$ , and  $\Delta C$ . By applying this approach to the coupled equations of heat transport and fluid flow, one obtains a system of equations for the  $i$ th finite element of the general form:

$$X_i = \left[ \begin{array}{cc} \int_{\bar{R}} \omega_j \frac{\partial \varepsilon_1}{\partial T} dR & \int_{\bar{R}} \omega_j \frac{\partial \varepsilon_1}{\partial h} dR \\ \int_{\bar{R}} \omega_j \frac{\partial \varepsilon_2}{\partial T} dR & \int_{\bar{R}} \omega_j \frac{\partial \varepsilon_2}{\partial h} dR \end{array} \right] \begin{Bmatrix} \Delta T \\ \Delta h \end{Bmatrix} + \begin{Bmatrix} \int_{\bar{R}} \omega_j \varepsilon_1 dR \\ \int_{\bar{R}} \omega_j \varepsilon_2 dR \end{Bmatrix} \quad (9)$$

The first term (in brackets) is the Jacobian matrix and the last term (in braces) is the load vector. For brevity, the above equation shall be expanded for the discrete domain only.

Substituting the appropriate expression for the residual error, the following components of the Jacobian matrix are obtained:

$$\int_L \omega_i \frac{\partial \varepsilon_1}{\partial T} dL = S_i \int_L \omega_j \frac{\partial}{\partial T} \left( \frac{\partial T}{\partial t} \right) dL + \rho c_t \int_L \omega_j q \frac{\partial}{\partial T} \left( \frac{\partial T}{\partial L} \right) dL + D_t \int_L \frac{\partial \omega_j}{\partial L} \frac{\partial}{\partial T} \left( \frac{\partial T}{\partial L} \right) dL \quad (10)$$

$$\int_L \omega_i \frac{\partial \varepsilon_1}{\partial h} dL = -\rho c_t K_t \int_L \omega_j \frac{\partial T}{\partial L} \frac{\partial}{\partial h} \left( \frac{\partial h}{\partial L} \right) dL \quad (11)$$

$$\int_L \omega_i \frac{\partial \varepsilon_2}{\partial T} dL = -\gamma \int_L \omega_j \frac{\partial}{\partial T} \left( \frac{\partial T}{\partial t} \right) dL \quad (12)$$

$$\int_L \omega_j \frac{\partial \varepsilon_2}{\partial h} dL = S_t \int_L \omega_j \frac{\partial}{\partial h} \left( \frac{\partial h}{\partial t} \right) dL + K_t \int_L \frac{\partial \omega_j}{\partial L} \frac{\partial}{\partial h} \left( \frac{\partial h}{\partial L} \right) dL \quad (13)$$

The expansion of the load vector yields the following components:

$$\int_{\mathcal{L}} \omega_j \varepsilon_1 dL = S_t \int_{\mathcal{L}} \omega_j \frac{\partial T}{\partial t} dL + \rho c_t \int_{\mathcal{L}} \omega_j q \frac{\partial T}{\partial L} dL + D_t \int_{\mathcal{L}} \frac{\partial \omega_j}{\partial L} \frac{\partial T}{\partial L} dL - Q \int_{\mathcal{L}} \omega_j dL \quad (14)$$

$$\int_{\mathcal{L}} \omega_j \varepsilon_2 dL = S_f \int_{\mathcal{L}} \omega_j \frac{\partial h}{\partial t} dL + K_f \int_{\mathcal{L}} \frac{\partial \omega_j}{\partial L} \left( \frac{\partial h}{\partial L} + \delta'_b \right) dL - \gamma \int_{\mathcal{L}} \omega_j \frac{\partial T}{\partial t} dL \quad (15)$$

This approach was also applied to the mass transport equation which yields finite-element equations in terms of the change of concentration,  $\Delta C$ . The above expressions are easily extended to the case of the continuum elements.

#### *Approximation and solution techniques*

The dependent variables in the governing equations are approximated using quadratic shape functions given by:

$$\Phi(x, y, t) = \sum_{i=1}^m \omega_i(x, y) \Phi_i(t) \quad (16)$$

where

- $\Phi$  = continuous variable,  $T$ ,  $h$ , or  $C$
- $\omega_i$  = shape functions
- $\Phi_i$  = nodal values of the dependent variables
- $m$  = number of node points on the finite element.

Fluid properties such as density and viscosity have been measured in the laboratory and tabulated<sup>14</sup> as functions of temperature; these data were fitted using splines to provide smooth interpolating functions. The integration of the finite-element equations is performed using Gaussian quadrature. The final system of algebraic equations for each time step is solved using a frontal solution technique.

## MODEL APPLICATIONS

Numerical results from application to various test cases are presented here, and demonstrate the general capability of the new finite-element techniques. The test cases consist of the following simulation problems: (1) flow through networks of discrete fractures, (2) steady flow in a porous medium with a single fracture, and (3) solute transport through a single fracture with diffusion into a porous matrix.

#### *Flow in fracture networks*

The problem of calculating fluid flow through a system of fractures is analogous to that of modelling a fully developed flow in an interconnected pipe network. A number of computational techniques from pipe flow analysis have, in fact, been used by various researchers to solve the steady-flow equations in fracture networks. Krizek *et al.*,<sup>15</sup> for example, have analysed fluid flow in various fracture sets using a link node finite-difference approach. Selected cases from their work are used here for comparison with the finite-element solutions.

Two particular cases analysed by these authors consist of fracture sets intersecting at 90 and 120°, respectively. Hydraulic heads are specified at the left and right boundaries and at one internal node point of the network. These boundary conditions impose a net hydraulic

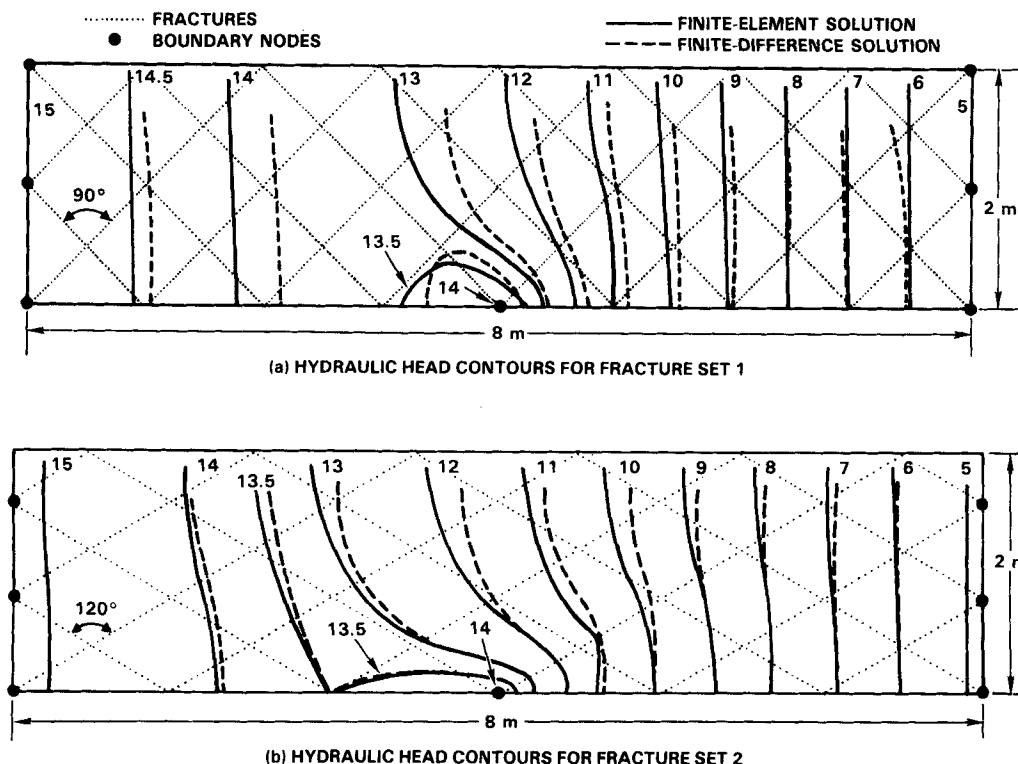


Figure 2. Comparison of finite-element and finite-difference solutions for flow in fracture systems

gradient from left to right. The fracture apertures in each system are uniform and are assigned a value of  $5 \mu\text{m}$ . Applying the power law from the Poiseuille flow equation, the assigned aperture is converted to a hydraulic conductivity of  $2 \times 10^{-3} \text{ m/s}$ .

For the two test cases, the isothermal flow equation was solved by using a network of line elements to represent the discrete fractures. A comparison of the finite-element results with those obtained by Krizek *et al.*<sup>15</sup> is presented in Figure 2. Both solutions for the hydraulic head distribution generally show very good agreement.

To demonstrate the validity of the line element approach, a third test case is considered. This test case, using experimental data, originally analysed by Wilson and Witherspoon,<sup>6</sup> consists of flow through an orthogonal network of conduits. The conduits are of equal length and diameter. The flow distribution through the network is established by inflow at nodes along the left boundary and outflow at one node point on the bottom of the network. Hydraulic head measurements were reported for the conduit intersections (nodes). Overlaying a network of 67 line elements, a steady-flow calculation was performed using specified head boundary conditions. A comparison of the measured and calculated head values is presented in Figure 3. The maximum relative difference in these values is about 2 per cent. These differences are probably of the same order of magnitude as the measurement error.

*Porous medium with single fracture*

For this test case, we consider the problem of isothermal steady flow in a porous medium with a single fracture embedded in the centre of the domain. The fracture orientation is

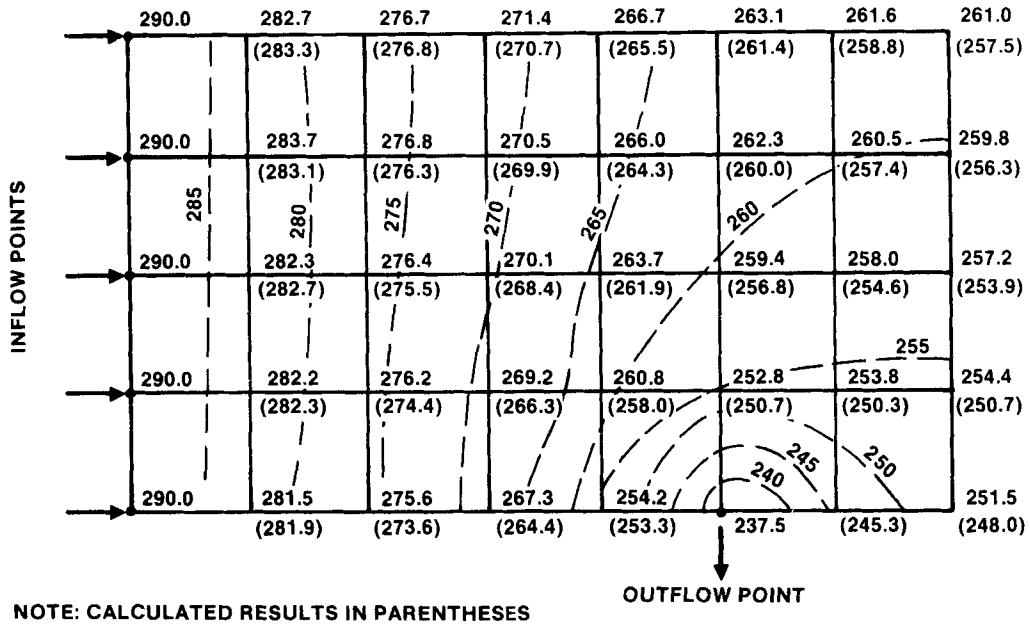


Figure 3. Measured and calculated heads for flow in conduit network

parallel to the general direction of flow, and the fracture aperture is selected so as to give a fracture permeability that is  $10^4$  greater than that of the porous medium. In this way, the fracture becomes the path of least resistance and, thus, will divert flow into and through the fracture. The high fracture permeability also has the effect of reducing the hydraulic gradient along the fracture.

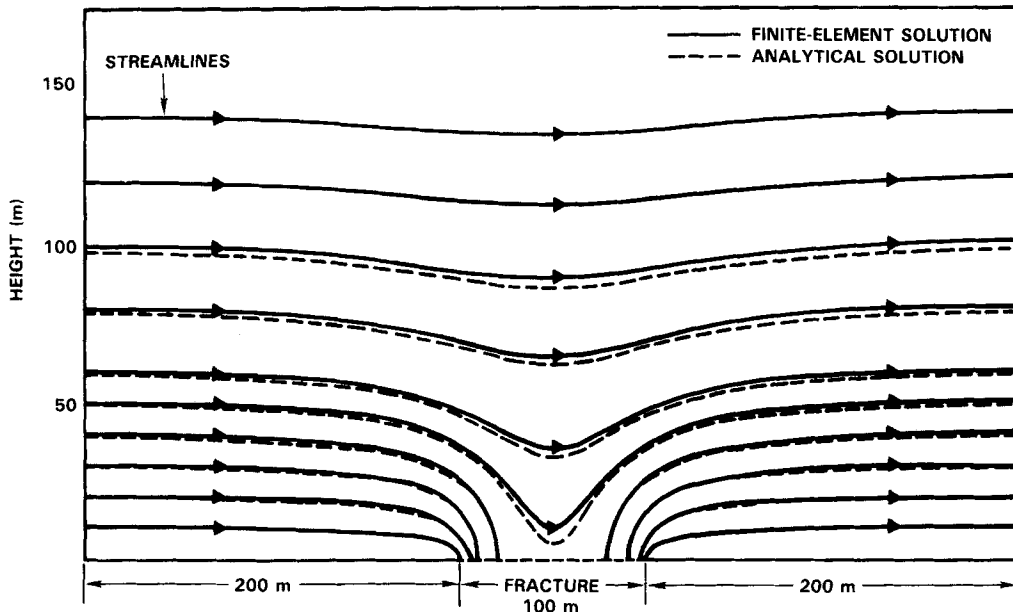


Figure 4. Comparison of analytical and finite-element solutions for flow in a porous medium with single fracture



At high-permeability contrasts, this fractured-porous medium problem very closely resembles a classic fluid flow problem from potential flow theory. In particular, the flow of an ideal fluid around a flat plate of finite length, where the free-stream velocity is perpendicular to the plate, exhibits a flow net that is analogous to the one obtained for the fractured-porous medium problem. The only difference between the two flow systems is that the role of potential,  $\phi$ , and stream function,  $\psi$ , is reversed (i.e. the  $\psi$ - $\phi$  flow net for the potential flow case corresponds to the  $\phi$ - $\psi$  net for the fractured-porous medium). The closed-form analytical solutions<sup>16</sup> available for the potential flow problem were used as a basis for comparison with the finite-element solution.

In specifying the test case, a rectangular domain 1000 m long by 200 m high was assumed with a 100-m-long fracture at the base of the region. The boundary conditions were selected to give a horizontal head gradient of  $10^{-3}$  m/m; no flow conditions were set on the top and bottom boundaries. A finite-element network was overlayed on the domain, consisting of 200 quadrilateral elements. Six line elements connected end to end were used to represent the fracture. Graphic comparisons of the analytical solution and the finite-element solution for the streamlines are shown for a subregion around the crack (Figure 4). Very close agreement between the two solutions is clearly indicated.

*Solute transport in a fractured-porous medium*

Applicability of the new finite-element technique to solute transport problems is demonstrated here with a simple test case. We consider a rectangular porous domain with a through-running horizontal fracture at the bottom boundary. The boundary conditions are specified so that flow and transport occur along the discrete unfilled fracture and only molecular diffusion (transverse to the flow direction) occurs in the porous rock. The geometry and boundary conditions for this test case are illustrated in Figure 5. Tang *et al.*<sup>17</sup> have developed a general analytical solution for this problem, where the solute is a radioactive tracer. Analytical solutions were obtained for the following choice of physical

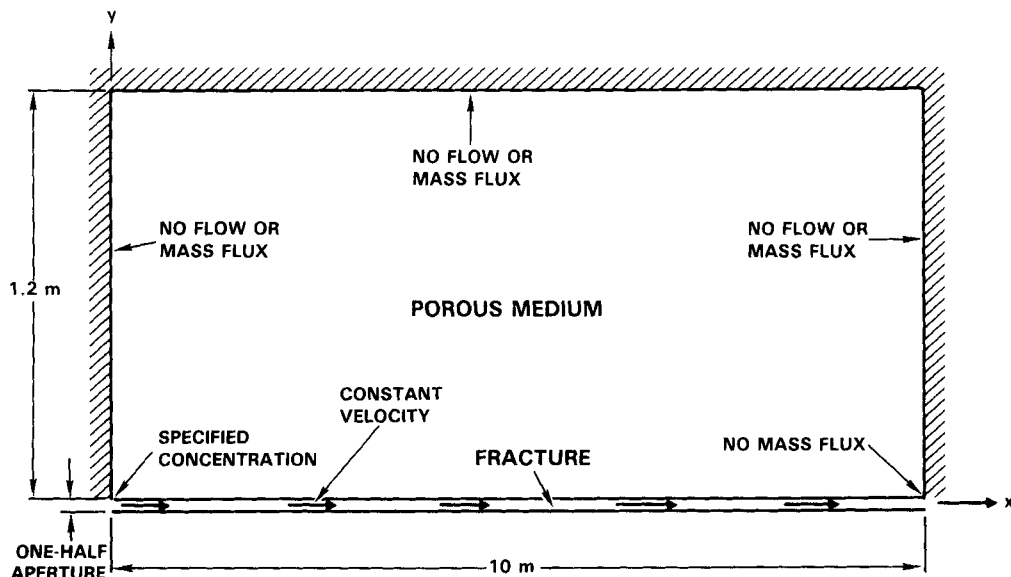


Figure 5. Physical setting and boundary conditions for solute transport problem

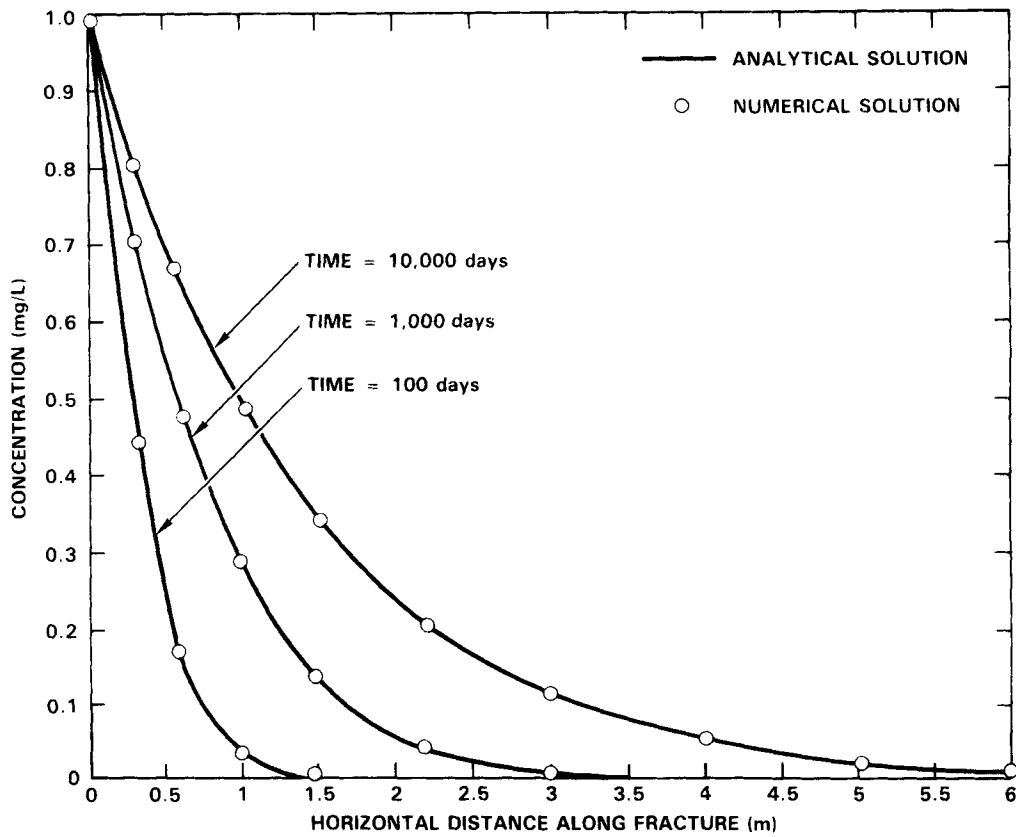


Figure 6. Analytical and finite-element solutions for solute transport in fractured-porous medium—concentration profiles along the fracture

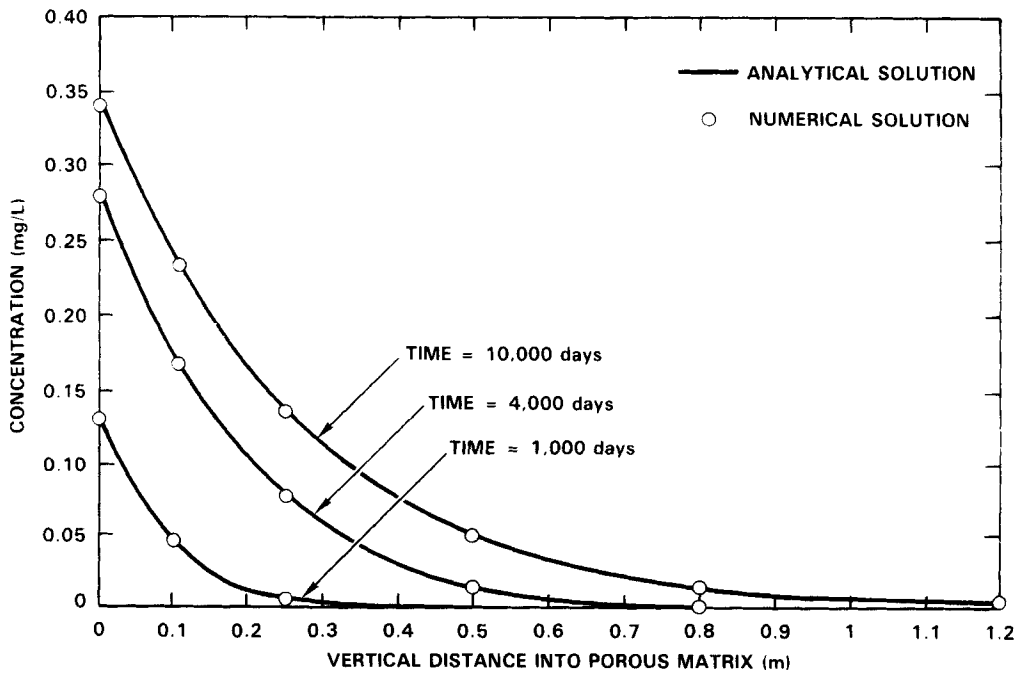


Figure 7. Analytical and finite-element solutions for solute transport in fractured-porous medium—concentration profiles through the porous matrix

parameters: (1) fracture properties are  $D_m = 5.95 \times 10^{-4} \text{ cm}^2/\text{s}$ ,  $q = 1.17 \times 10^{-5} \text{ cm/s}$ ,  $e/2 = 5 \times 10^{-3} \text{ cm}$ ; and (2) porous matrix properties are  $D_m = 1.6 \times 10^{-6} \text{ cm}^2/\text{s}$ ,  $\phi = 0.01$ . The solute is assumed to have a half-life of 12.35 yr and to be non-sorbing.

To obtain finite-element solutions for this problem, a rectangular mesh was used to represent the domain, which consists of 70 quadrilateral elements and 14 line elements. The line elements were embedded along the bottom boundary of the network. The transient solution was calculated using variable time steps ranging from 20 to 1000 d. The finite-element solutions are compared with the analytical solutions in Figures 6 and 7. These graphic comparisons indicate very good agreement between results.

## CONCLUSIONS

A new approach to the problem of modelling fluid flow in fractured-porous media has been developed and demonstrated, and is based on the use of a two-dimensional finite-element technique and the principle of superposition. This new approach can easily be incorporated into existing finite-element models for porous media flow to extend their capabilities: (1) to accommodate simultaneous flow in porous strata and fracture systems and (2) to model groundwater flow in strata-form geologies where large contrasts in layer thickness exist. The approach can also be extended to three-dimensional fluid flow problems by introducing two-dimensional plate elements to represent discrete hydrologic features. Moreover, the approach is also applicable to the problem of modelling solute transport in fractured-porous media.

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